Linear Inequalities

- Two real numbers or two algebraic expressions related by the symbol ‘<’, ‘>’, ‘≤’ or ‘≥’ form an inequality.

  Note: Inequalities involving ‘<’ or ‘>’are strict inequalities whereas inequalities involving ‘≤’ or ‘≥’ are slack inequalities.

  Example: $6 < 26$, $3 < z + 1 \leq 22$, $27 \geq s \geq 16$, $p + t > 100$

  Example 1:A shopkeeper sells some television sets and A/C sets every day. He wants to earn a profit of at least Rs 10000 each week. For this, he must sell 20 television sets and 30 A/C sets every week or he must sell 50 television sets and 10A/C sets every week. Express this situation mathematically.

  Solution: Let the profit on each television set be Rs x and the profit on each A/C be Rs y.

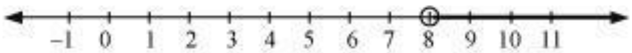
  It is given that to earn a profit of at least Rs 6500, the shopkeeper should sell 20 television set and 30 A/C sets every week. Hence, the total profit earned by the shopkeeper by selling 20 television sets and 30 A/C sets is Rs $(20x + 30y)$. Thus, $20x + 30y \geq 10000$
  In either condition, it is given that to earn a profit of at least Rs 10000, the shopkeeper should sell 50 television sets and 10 A/C sets every week. Hence, the total profit earned by the shopkeeper by selling 50 television sets and 10 A/C sets is Rs $(50x + 10y)$. Thus, $50x + 10y \geq 10000$.
  Hence, the given situation can be expressed mathematically by the following two inequalities:
  $20x + 30y \geq 10000$ and $50x + 10y \geq 10000$

- Any **solution of an inequality in one variable** is a value of the variable that makes it a true statement.
- The set of numbers consisting of all the solutions of an inequality is known as the **solution set** of the inequality.

  The rules that need to be followed to solve an inequality are:
  - Equal numbers may be added to (or subtracted from) both sides of an inequality without affecting the sign of the inequality.
  - Both sides of an inequality can be multiplied (or divided) with the same positive number. However, when both sides are multiplied or divided by a negative number, then the sign of the inequality is reversed.
- To represent $x \leq a$ (or $x \geq a$) on a number line, encircle the number a, and darken the line to the left (or the right) of a.

Example: Show the graph of the solution of the inequality $5(x - 3) > 2x + 9$ on number line.

Solution: $5(x - 3) > 2x + 9$
$\Rightarrow 5x - 15 > 2x + 9$
$\Rightarrow 5x - 15 - 2x > 2x + 9 - 2x$
$\Rightarrow 3x - 15 > 9$
$\Rightarrow 3x > 9 + 15$
$\Rightarrow 3x > 24$
$\Rightarrow x > 8$
Thus, the solution of the given inequality can be represented on the number line as shown below.



- The solution set might be taken from real numbers or whole numbers or integers or any other set of numbers. The set from which the values of the variables (involved in the inequation) are chosen is called the **replacement set**. We may take any set as the replacement set. For example, **N**, **Z**,{−4, −3, −2}can be taken as the replacement set.

- **Linear inequalities in two variables and representing their solution graphically**

Rules for solving an inequality:

  - Equal numbers may be added to or subtracted from both sides of an inequality without affecting the sign of the inequality.
  - Both sides of an inequality can be multiplied with or divided by the same positive number. But when both sides are multiplied with or divided by a negative number, the sign of inequality is reversed.

Example 1: Solve $3\left(\frac{3}{5}x + 4\right) \geq 2(x - 3)$.

Solution:
$3\left(\frac{3}{5}x + 4\right) \geq 2(x - 3)$

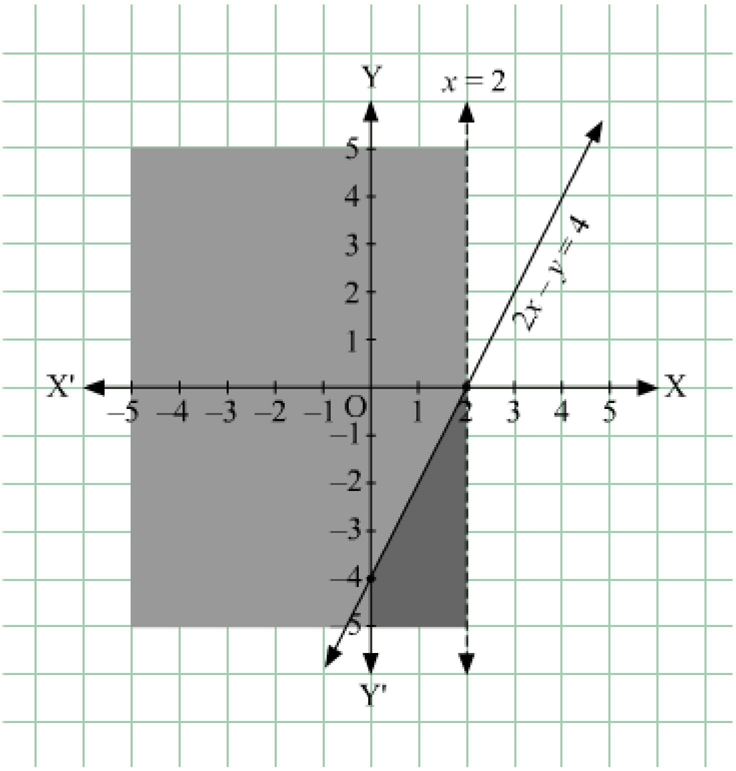
$\Rightarrow 3\left(\frac{3x + 20}{5}\right) \geq 2(x - 3)$
$\Rightarrow 3(3x + 20) \geq 10(x - 3)$
$\Rightarrow 9x + 60 \geq 10x - 30$
$\Rightarrow 9x - 10x \geq - 30 - 60$
$\Rightarrow - x \geq - 90$
$\Rightarrow x \leq 90$
∴ The solution set of the given inequality is $(- \infty, 90]$.
The solution region of a system of inequalities is the region which satisfies all the given inequalities in the system simultaneously.
In order to identify the half plane represented by an inequality, it is sufficient to take any point (a, b) (not on the line) and check whether it satisfies the inequality or not. If it

satisfies, then the inequality represents the half plane containing the point and we shade this region. If not, then the inequality represents the half plane which does not contain the point. For convenience, the point (0, 0) is preferred.
In an inequality of the type $ax + by \geq c$ or $ax + by \leq c$, the points on the line $ax + by = c$ are to be included in the solution region. So, we darken the line in the solution region.
In an inequality of the type $ax + by > c$ or $ax + by < c$, the points on the line $ax + by = c$ are not to be included in the solution region. So, we draw a broken or dotted line in the solution region.

Example 2: Solve the following system of linear inequalities graphically: $2x - y \leq 4$, $x < 2$

Solution: The given linear inequalities are
$2x - y \leq 4$ ... (i)
$x < 2$ ... (ii)

The graphs of the lines $2x - y = 4$ and $x = 2$ are drawn in the figure below.



Inequality (i) represents the region on the left of the line $2x - y = 4$ (including the line $2x - y = 4$). Inequality (ii) represents the region on the left of the line $x = 2$ (excluding the line $x = 2$).
Hence, the solution of the given system of linear inequalities is represented by the common shaded region, including the points on the line $2x - y = 4$.